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Scaling and Learning in Nuclear Energy (Presented at the Workshop on Size and Productive Efficiency: The Wider Implications, IIASA, 25-29 June 1979)

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PREFACE

This paper was presented at the workshop "Scale and Productive Efficiency: The Wider Implications" held at IIASA in June 1979. Because of its relevance to understanding the increase in scale of nuclear power plants and the role of learning it is being published as a collaborative paper in order to give it wider circulation.

CONTENTS

SCALING AND LEARNING IN NUCLEAR ENERGY

B.I. Spinrad

INTRODUCTION

Scaling is necessarily a function of learning. At any given time, there is a size above which we can't build. Some of the reasons are external--a limited market, for example--but some are clearly internal. We don't know how to build it, as with steel pressure vessels above a certain size and pressure capability; or, if we think we know how to build, we sometimes don't know how to do it economically--as with sodium-to-steam heat exchanges of large capacity. Sometimes, the connection of risks in one package gets too big to handle--thus the imposition of a capacity limit on light-water reactors (LWR) in the United States. And there are doubtless other reasons for things being "too big to build". All these limitations change with time. We can learn how to avoid or overcome them, or in some instances to ignore them as they turn out to be mental rather than physical limits.

Often, the most significant advances, in "learning how to do it" and in finding an appropriate scale for a standardized product, occur when the technology is new, uncertain and plastic. But the strategy of market penetration is itself a factor in learning how to build what size plant. The reason is that early embodiments of a new technology are rarely economic in them-sevles. They are to some extent practice installations, variously labeled as "large pilot plants", "demonstration units", "experimental plants", "pioneer plants" or "prototypes". There is a tension between two objectives of building such units: to achieve economic competitiveness as soon as possible; and to minimize the expense of building and operating non-economic plants. The first objective emphasizes the positive aspects of scaling and introduces a tendency toward rapid growth in scale of these plants. The second objective emphasizes the positive aspects of learning and introduces a tendency to build minimum-size units. Intuitively, one feels that there is an optimal trajectory of scale increase under the circumstances. Studying this trajectory is a topic for planned research; but it is certain that the interaction between scaling and learning is a necessary input to that research. And, because this interesting phase of commercialization is my particular target, the focus of attention here is on this early period--the period of rapid scaling and quick learning.

It goes without saying that I am talking about industrial systems here. There are products--hand-held calculators, toys, bicycles--whose scale is rigidly circumscribed by the product application. They form a class of systems for which the learning curve can be applied in almost pure form. But even within industrial systems, there are types of plants for which scale is circumscribed by a limited market. For example, it would do little good for a manufacturer of power-plant turbines to scale his facilities to production of 100 units of, say, 3000 MW-of-heat capacity, per year. It is technically possible, and it may even have demonstrably lower unit cost, but the market just isn't there (at least, now). Thus, we are concerned with industrial plants embedded within an industry large enough so that many plants, even though they might be of very large size, are needed to saturate the market. This is fairly common in basic industries. In the following discussion, we shall be considering the electrical power generating industry, which meets this criterion. Specifically, we shall be taking our examples from, and directing our discussion toward, the nuclear component of the electrical industry. Two types of system--the light-water reactor (LWR) in the United States and the heavy-water reactor (CANDU) in Canada--provide illustrations of past trajectories. A third type of system, the liquid-metal-cooled fast breeder reactor (LMFBR) is being developed in several countries and is a proper subject for projective study.

We reserve detailed analysis for our application study, to be presented later, and here present a very condensed and qualitative (<u>and</u> subjective) history of the development of these nuclear systems.

Light water reactors are the dominant nuclear systems of the world today. They had their genesis from the use of light (ordinary) water reactors in the United States for three purposes: production of nuclear weapons material (at Hanford); reactors for research purposes (Oak Ridge National Laboratory); and submarine naval propulsion (Argonne National Laboratory and Westinghouse-Bettis). The pressurized water reactors used for naval propulsion are actually conceptually derived from research reactors. Today's LWR's still bear traces of all three ancestors: fuel design from the naval program, neutronic design and construction technology from the production program and control and safety design from research reactor experience.

The first embodiments of LWR's for power were the naval propulsion reactors, from which the pressurized water reactors

(PWR) were descended, and experimental boiling water reactors (BWR) developed at the US National Laboratories. The first continuously operable power reactors were among these latter, BORAX-II (3 Mwe, maximum), commissioned in 1955, and EBWR (5 MWe maximum) commissioned in 1956. (In what follows, dates are year of commissioning.) The first power reactor designed for long-time commercial operation was the Shippingport PWR (60 MWe, 1957). The first privately ordered and commissioned power reactors were Indian Point #1 (PWR, 265 MWe, 1962), Dresden 1 (BWR, 200 MWe, 1960) and Yankee-Rowe (PWR, 175 MWe, 1961). There followed a period of relatively slow growth in commercial orders, which came to an end with the ordering and commissioning of Connecticut Yankee (PWR, 575 MWe, 1968) and Oyster Creek #1 (BWR, 620 MWe, 1969). These were built at a scale which was considered fully commercial at that time. Subsequently, a combination of scaling application, vendor initiative and simple growth mentality produced a flood of nuclear orders of increasing plant capacity, which lasted from 1967-1972. This came to a stop when a combination of diminished growth potential for the electrical industry in the US, the appearance of organized political opposition to nuclear power, and a disintegration of the political and industrial infrastructure supporting LWR's appeared in the early 1970's. The nuclear industry of the US has been moribund since that time. Its continued existence is largely due to the spinning off of LWR sales into foreign markets, achieved variously by sale of reactors, of licenses to manufacture according to US experience, or simply through emulation by and partnership with firms abroad. The continued growth of the nuclear-electric supply industry in the United States is almost entirely due to delayed, stretched-out delivery of orders placed in the early 1970's.

The CANDU reactor has had a much more goal-oriented history. Its genesis was the first Canadian research reactor, NRX. In spite of the occurrence of a major melt-down of that reactor in 1950, the experience developed with it was encouraging to the further marriage of heavy-water and PWR technology. An experimental reactor, NRU, and a pilot-plant system, NPD, followed. Favorable operating experience and economic prospects resulted in the commissioning of successive generations of power plants. The first, Douglas Point, had a power of 206 MWe and was commissioned in 1968. After that time orders for several-unit stations going from Pickering #'s 1-4 (515 MWe, 1971-1973) through Bruce #'s 1-4 (740 MWe, 1977-1979), Pickering #'s 5-8 (515 MWe, 1983-1986), Bruce #'s 5-8 (756 MWe, 1983-1986) and Darlington #'s 1-4 (881 MWe, 1986-1988). CANDU reactors have also been sold abroad, notably to India (RAPP, 202 MWe, 1973) which has since been making its own and to Pakistan (KANUPP, 125 Mwe, 1972). Competitive or variant versions of heavy water power reactors have also been developed, but not to the point of market penetration. These include British, German, and Swedish systems as well as two alternate Canadian designs.

The LMFBR has been under long development. Early experimental reactors in the US and USSR were followed up by further, larger experimental reactors in the UK and France. A pilot-plant demonstration, EBR-II (17.5 Mwe, 1963) and a prototype, Enrico Fermi #1 (57 MWe, 1965) were built in the early 1960's in the US, but these were not followed up by further commercialization. At present, France and the USSR, with successful operation of large prototypes (PHENIX, 233 MWe, 1973; BN-350, 350 MWe equivalent, 1973) are the countries most advanced toward the commercialization phase.

A listing of most power reactors of the world, from which more summary data can be derived, is found in reference [1].

THE SCALING LAW

In the construction of large industrial facilities, engineers are taught to expect economies of scale: that the bigger the plant, be it to make electric power, refined oil products, steel, chemicals or clothing, the more economical the product will be. As a heuristic generalization for this, it is pointed out that the plant capital cost is commonly determined by the mass of material, and this is roughly proportional to the surface of the buildings and equipment; whereas capacity is determined to first approximation by plant and equipment volume. Since surface is proportional to volume taken to the two-thirds power, we then can justify an approximate relation:

$$\text{Capital Cost} = \text{constant} \times (\text{plant output rate})^{2/3}$$

Under these circumstances, the cost per unit product might be expected to decrease with the plant capacity according to:

$$\frac{\text{Cost}}{\text{Unit product}} = \text{constant (plant output rate)}^{-1/3}$$

Other heuristics can be invoked to indicate trends of costs with scale:

-- An engineer can sketch out a large project, as a physical entity, as easily as he can a small one.
-- When a component gets large enough, we no longer can be sure we can build it.
-- When a project gets complex enough, management problems increase dramatically, and labor productivity may decrease.
-- Big plants take longer to build than smaller ones, thereby incurring larger interest payments during construction.
-- Systems with many components are less reliable than those with fewer components.
-- Big pieces take more time to repair than little ones do.
-- And many more.

These heuristics are only indicative, and they are not all in the same direction. Some considerations (e.g., engineering cost) would appear to indicate a constant cost regardless of size--a decrease of unit cost as the reciprocal of plant size;

complexity, on the other hand, is a faster-than-linear function beyond some plant size, and would tend to make the curve go up. Nevertheless, the standard scaling law is written as:

$$C = AP^{-\beta} \quad , \tag{1}$$

where C is cost per unit plant capacity, P is plant capacity, A is a constant of normalization and β is a "scaling parameter". The surface-to-volume heuristic is usually expected to be the dominant one, corresponding to a value for β of 1/3. A surprising number of cases do, in fact exhibit β values close to 1/3 over plant sizes of interest.

Equation (1) has obvious limitations. As $P \to 0$, $C \to \infty$, for example. This is not quite intuitive. If C approaches a limit for small P, a trivial change can fix equation (1), however. Inclusion of a small constant λ to make the law into

$$C = A(P+\lambda)^{-\beta} \tag{2}$$

lets the result converge as $P \to 0$ without changing the law for large P. Since we will be interested in plants of increasing size, we will not, however, refer to this law further.

At the other end of the scale we get a different problem. Some of the heuristics that <u>increase</u> unit cost with size can become dominant. We have already indicated that large size can increase the costs of project management and possibly decrease plant reliability. These introduce correlated expenses due to increased construction time (e.g., increased financing costs), loss of productivity of construction labor, and low capacity factor for the completed plant. As components get larger, more construction takes place in the field, and field construction is intrinsically more expensive than is factory construction and assembly. (The break-point may be thought of as the point at which a project can more properly be described as "field-constructed" than as "erected"). The problem of technological limits to the scale that can be reached has also been mentioned.

All of these considerations point to the fact that, at any given time, there must be a plant size at which unit cost becomes a minimum. The scaling law in its primitive form, equation (1), is no longer usable.

How to deal with this situation is a primary target of this paper, and we will return to the point later.

THE LEARNING CURVE

Heuristically, the learning curve is simpler than the scaling law. It is a formalized way of expressing the qualitative thought that the more experience we have, the more efficient we become.

As with so many other human phenomena, but particularly by analogy with psychological time--which expresses the thought that any individual perceives time duration essentially as a fraction of life experience--we tend to measure production experience logarithmically. We learn the same amount with each doubling of product output; or so goes the reasoning. Thus, the scaling law may be exhibited in one of two forms. The first characterizes learning as an absolute cost reduction proportional to the logarithm of output, i.e.

$$C(n) = C(1) - a \ln n \quad , \qquad (3)$$

where C is a function of the number of units produced so far, denoted by n.

The second uses a relative cost reduction as the measure of efficiency, giving

$$C(n) = C(1) n^{-b} \quad . \qquad (4)$$

It does not, as does (3), become negative as $n \to \infty$, for which reason we will adopt (4) as the standard learning curve law. Note that (4) is well approximated by (3) for small b, if a is defined as b C(1).

There are no difficulties in applying (4) at the early stages of production, but as time goes on two problems arise: one is formal, the other phenomenological.

The formal problem with equation (4) is that it appears to take the maxim "practice makes perfect" too seriously. Although C is always positive, it goes to zero for large n. If only because costs of materials are irreducible, there must be a limit to how far down cost can be driven. Also, production rates must approach a finite limit, implying capital costs that are not zero, and we also sense that there are limits to labor productivity. Lunch may be cheap, but it is never free. C must approach a non-zero asymptote.

The phenomenological problem is that production takes place in a socio-economic environment. Inherent in a dynamic society is the notion of change in that environment. Specifically, the relative costs of inputs to the production process change. A material may become scarcer, labor may command rates which escalate at a different pace from productivity, and so on. Our recent experience has illustrated the shifting cost of the important input, energy. The response of an alert production system to these shifting input costs is a reoptimization of input requirements. This reoptimization is, of course, part of the learning process. However, the impact on unit cost is still felt, even after reoptimization is achieved. The impact is basically a renormalization of the cost curve, even when it is expressed in constant-value currency. There is an extrinsic time factor involved, in addition to the intrinsic accounting of time by the measure of n.

The learning curve is the natural rule of thumb to use when considering costs of mass-produced items. Nevertheless, there is no reason why it should not apply to arrays of industrial equipment, e.g. factories, as well. However, as contrasted to the scaling law, it is usually considered to be essentially institutional. That is, the learning curve would be expected to be experienced according to the production of a particular manufacturer, rather than by total production of all of a group of competitors. To be sure, there is cross-talk within an industry, usually more than industrial-secret minded managers even dream. Thus, we might expect that a perfect learning curve would show primary dependence on, not one, but two measures of production: n_k, the production experience of the k'th producer, and $N \equiv \sum n_k$, the total production experience of the industry. A further limitation of equation (4) is, then, that it is both (or either) vague and/or parochial with regard to the definition of production experience.

MODIFIED LAWS

Both the scaling law and the learning curve are approximations, valid for only a limited time, capacity range, production period, and so on. We now go about "fixing up" these laws to make them, at least in principle, more adaptable. This will, of necessity, increase the number of parameters with which we must deal. The resulting curves may be more easily fitted to data from the real world; but in fact, this remains an experience in pure modeling, and we must be on guard against facile causal explanations of what amounts to a simple study in correlation.

We have noted that the scaling law must give out beyond some plant size, and illustrated several heuristics to explain this. We now give these mathematical form. What we are attempting to do, of course, is to describe properties of the cost-capacity curve which arise when we approach or exceed an optimal capacity. For this, at least, two extra parameters are needed: one to determine the cost-minimum capacity, and another to determine what the cost is at that minimum. There are, of course, an infinite number of ways to select parametric curves of this type.

The form chosen is

$$C = AP_{\mathrm{Min}}^{-\beta}\left[\left(\frac{P}{P_{\mathrm{Min}}}\right)^{-\beta} + \delta\left(\frac{P}{P_{\mathrm{Min}}}\right)^{\beta/\delta}\right] \quad . \tag{5}$$

The choice of (5) makes it easier to identify the parameters phenomenologically. P_{Min} is the capacity or size at which C is minimum. δ is the fractional increase in unit cost which is exhibited, at P_{Min}, over the unit cost which would be expected if the simple scaling law, equation (1), were correct. Many other forms could be invented, but any which have the same values of P_{Min} and δ as equation (5) will show very similar behavior.

The other modifications which we make in the scaling law is to identify the four parameters, A, β, P_{Min} and δ, as functions of time. That is, we may write (5) in the explicit form

$$C(P,t) = A(t)\left[P_{Min}(t)\right]^{-\beta}\left[\left[\frac{P}{P_{Min}(t)}\right]^{-\beta} + \delta(t)\left(\frac{P}{P_{Min}(t)}\right)^{\beta(t)/\delta(t)}\right] . \tag{6}$$

We now ask which of these parameters vary either significantly with time or strongly with time.

A(t) can vary strongly with time. It is the normalizing factor on unit cost, and can be expected to be subject to some sort of learning factor. On the other hand, for most applications of the scaling law, A is not a significant parameter. This is because the question addressed by examining the scaling law is "What size plant should be built at a given time?" The decisions for which A is important are decisions as to whether to build a plant of one type or another; but such decisions rest only to a limited extent on economies of scale.

β can be expected to vary slowly with time. It has already been noted that β is determined by the types of economies which result from increasing the size of small plants. These are dominated by the surface-volume paradigm (β around 1/3) and the cost-regardless-of-size paradigm (β around 1). There are also other factors, which scale differently, even for small plants. The main point about β is that it is a fitting parameter to express a linear sum of terms, and that the terms which are not dominant are large in number. Under these conditions, the average change in trend as circumstances change is small.

The parameter which is both strongly and significantly variable with time is P_{Min}. All of the factors which increase the unit cost of large size plants are subject to learning. We can learn how to increase the size and complexity of factory-built components; how to improve schedules by contract and work-force management; how to design for more efficient servicing and shorter and less frequent down-time; and so on. Of course, conceptually it seems appropriate to consider that learning in all these categories is subject to some law of diminishing returns. But it seems logical that P_{Min} should increase with time, although the rate of increase slows. This suggests that P_{Min} is a parameter that might be itself represented by a learning curve.

Finally, we have little to guide us with regard to the behavior of δ. This parameter tells us how sharp the cost minimum is, among other things; for

$$\left.\frac{C''}{C}\right|_{P_{Min}} = \frac{1}{\delta}\,\frac{\beta^2}{P_{Min}^2} \quad . \tag{7}$$

Thus, when δ is small, the cost minimum is relatively sharp and when δ is large it is rather broad. Figure 4 illustrates curve shapes for systems of different δ and two values of β. It might be noted among other features of the curves in Figure 4 that, below P_{Min}, the deviation of the cost curve from the form of equation (1) (the simple scaling law) is greater for large δ, and an evaluation attempting to fit the data to equation (1) might infer a good fit for a smaller β value than really pertains.

As to time behavior of δ, I find little guidance in heuristics. It seems reasonable that whatever learning there is that permits P_{Min} to increase will also act to decrease costs as P goes beyond P_{Min}. For that rationalization, but primarily because of its greater mathematical tractability, I have chosen δ to be a slowly varying function of time. Then, one can write to first approximation

$$\begin{aligned} C(P,t) = A(t)\left[P_{Min}(t)\right]^{-\beta}&\left(\left[\frac{P_{Min}(t)}{P}\right]^{\beta}\right.\\ &\left. + \delta\left[\frac{P}{P_{Min}(t)}\right]^{\beta/\delta}\right) \quad . \end{aligned} \tag{8}$$

It is assumed in (8) that β and δ, although they are in the long run also functions of time, can be held to be constant over a considerable period.

The learning curve is far easier to modify. It only remains to postulate an asymptotic value for unit cost--the intrinsic value of materials, for example. One can then write

$$C(n) = C(1)n^{-b} + C\infty \quad , \tag{9}$$

where $C\infty$ is that asymptotic value.

HYPOTHESIS

We have noted that scaling is a function of learning--an observation that is in no sense original. We are now in a position to formalize this as a hypothetical connection between the two rules*.

The essential hypotheses are:

1. The movement of the parameter P_{Min} in equation (8) is essentially governed by a learning curve.

2. The economies of the n^{th} plant, seen in the learning curve (equation (9)) are economies of scale as we proceed along the envelope of P_{Min}.

Hypothesis (1) can be formalized in the form that $P_{Min}(t) = P_{Min}(n[t]) \equiv P_m(n)$, as

$$P_m(n) = kn^{\gamma} \quad . \tag{10}$$

We now assume that the standard plant of sequence number n is built at $P_m(n)$. The cost at $P_m(n)$ is

$$C(P_m[n]) = A(1+\delta)k^{-\beta}n^{-\beta\gamma} \quad . \tag{11}$$

From the learning curve itself, however, we have

$$C(n) = C(1)n^{-b} + C\infty \quad . \tag{9}$$

Having just inserted the $C\infty$ in the previous section, we now ignore it, at least for industries under active product development.

Hypothesis (2) for $C\infty = 0$ can then be stated as

*This equivalence has previously been noted by Sahal [2] in a very generalized form and the results cited here are a specialization of his principle of self-similarity.

$$C(n) = C(P_m[n]) \quad , \tag{12a}$$

$$A(1+\delta)k^{-\beta}n^{-\beta\gamma} = C(1)n^{-b} \quad . \tag{12b}$$

Then, we can identify terms in (12b) so that

$$A(1+\delta)k^{-\beta} = C(1) \quad , \tag{13a}$$

$$\beta\gamma = b \quad . \tag{13b}$$

Equation (13a) is essentially a tautology. (13b) provides, however, a means of estimating γ; for, if we attempt to summarize scaling by examining experience with ever-larger plants, we estimate β, and if we attempt to summarize learning from the same data we estimate b. Finally, we can estimate γ directly by examining plant sizes.

Classical Values

The usually estimated values for both β and b are both 1/3. That is, a variety of studies of different fields have shown that there is a surprising tendency for the data to fit to these "classical" values. In the absence of study, they become the standard guess. This implies that the value of γ is near unity, or that the capacity of the new plant will tend to increase linearly in n.

This general rule should not be considered as contradictory to the other rule that new (and supposedly most economical) plant tends to be of a capacity which is a constant fraction of the total industrial capacity. Early in the industry, new plant is dominant, and the ratio of plant size to industry size is large. Initially, it decreases as 1/n, approximately. But after a while, old, smaller plants are decommissioned, and only the last "k" plants are really important. The ratio of new plant size to industry size then approaches the ratio 1/k.

Nuclear Industry Values

The nuclear industry does not appear to be a classical industry. The value of β is unusually large, corresponding to the fact that a large fraction of small-plant costs go into engineering services, licensing activities and land-preparation. These are activities whose costs are, to first approximation, invariant with plant size, and push β from its classical value of 1/3 in the direction of unity. The value of β actually seems to be of the order of 0.5, and it is this high value which led the early nuclear industry in the direction of very rapid scaling.

There are less data to indicate what the value of b is, but it is probably lower than 1/3. The nuclear industry tended to standardize power reactor construction practices earlier in time than most new industries. Moreover, these practices were not rapidly evolving ones: they were in almost every case adaptations of practices from the fossil-fueled power plant industry, with regressions in some performance criteria (for example, steam temperature). The fossil power plant industry is an old one, at the end of a long learning curve. Finally, the severe pressure of anti-nuclear agitation has enforced a degree of conservatism* on nuclear design practice which has very much interfered with innovation. There are some signs that, as a result, b is negative for some periods of time, even when adjusted for construction-industry inflation. Over the long run, it is however undoubtedly small and positive.

Thus, γ is probably closer to 0.5 than to unity, and may be even smaller. The consequence is that plant size should be growing only slowly.

If we take 60 MWe as the size of the first LWR (Shippingport) and the round number of 100 plants commissioned so far, $\gamma = 0.5$ would lead to 600 MWe as the standard size now. The actual rating of the standard LWR is now 1200 MWe but there are indications that this is larger than optimum, and indeed by perhaps a factor of 2 [3]. We could not anticipate 1200 MWe being the optimum until of the order of 400 plants have been built. This will happen by 1990 according to present construction schedules, and it is notable that these plants on order are virtually all 1200 MWe or less. Conversely, if 1200 MWe were the optimum, now, γ would have the value 0.65, and the optimum plant in 1990 would be almost 3000 MWe.

FINDING THE PARAMETERS

If one has a parametric law of engineering systems, and the parameters are not derivable from first principles, they are normally found by least squaring the data that exist. In so doing, other functional dependences that are extrinsic--in the case of economic scaling and learning, such time- or location-dependent variables as differences in economic setting (i.e., variable physical conditions, local regulations, etc.)--are also included in a regression analysis, so that if their effects are significant they can be corrected for. The corrected data are the data of "pure" scaling and learning. That is, they represent the best set of "ceteris paribus" data one can develop from experience.

Such data are rarely smooth. When plotted on log-log paper--again, a common engineering practice--they might resemble

*This conservatism results from the effects of public pressure on the licensing process. Both regulatory agencies and the industry are reluctant to "rouse the animals" by considering significant changes.

Figure 1, for a scaling system. Data of this quality are often experienced, and are usually treated, not by sophisticated data analysis, but by the "data band and central tendency" lines also drawn on Figure 1. The justification is that any departure from non-linearity which is not visible is likely to be reflected as an insignificant higher-order term in a regression analysis.

An a priori estimate of scaling behavior can also be made, and indeed is more in the spirit of "other things being equal". An experienced designer (or design team) is instructed to design plants differing from each other only in plant capacity. Then this, or a different, team is asked to prepare a consistent set of ground rules for costing, and to estimate the cost of each plant. The resulting set of cost estimates is likely to look like Figure 2. The points show less scatter than a posteriori plots, for obvious reasons; however, it is common to see discontinuities within the general trends, as size-limited subsystems or technologies may be optimum only within some range of sizes.

Both Figures 1 and 2 exhibit bands within which all points are located, and a central line showing the main trend. Both figures have also been intentionally drawn to be deceptive; one might call them malicious illustrations of the tendency to look at figures as straight lines. In fact, both curves were drawn from credible plots which scattered originally around a curve with a minimum. In Figure 1, the minimum was near the abscissa of the third point from the right; in Figure 2, the minimum was just beyond the farthest point to the right. We now take up the matter of discovery of such "hidden minima".

Figure 3 presents some of the curves which can be drawn according to equation (8) for $\beta = 0.3$. It can be noted that when δ is small--less than 0.3--the curves to the left of $P/P_{Min} = 0.1$ are virtually identical; and that when δ is very small--0.1 or less--the minimum is quite sharp. When δ is large, the minimum is very shallow, and even the logarithmic slope of the curve at $P/P_{Min} = 0.1$ deviates somewhat from the asymptotic value for small P.

Figure 4 presents curves for a more restricted range of P: from $P/P_{Min} = 0.1$ to $P/P_{Min} = 3$. This is about as large a range over which good data for industrial plant costs could be assembled at any given time. Typically, data for very small plants, decades smaller than commercial plants, pertain to pilot or experimental systems or to very old ones; it is very rash to try to correct such data to simulate small plants that would be typical of current industrial practice. Indeed, in many system types, and specifically for the case of nuclear power plants, the range over which usable data can be procured is even less than Figure 4's factor of 30.

Visual inspection of Figure 4 reveals that:

-- For $\beta = 0.3$ the minima are extremely flat for a factor of 2 in P near P_{Min} (i.e., from $P/P_{Min} = 0.7$ to 1.4).
-- If one were examining data below $P/P_{Min} \doteq 1.5$, it would be tempting to fit them to a straight line on the plot (i.e., a simple power law) whose slope is less than 0.3 in absolute value.
-- For $\beta = 0.5$ the minima are much sharper. Only for large δ would the curves suggest the possibility of a simple power law with small β.

Figure 4 has a point on it that illustrates an error bar of ±15%. This is a fairly consistent mean scatter of costs about estimates, and is thus the measure of how far real data points might scatter around the theoretical curves drawn. Figure 1 is, in fact, derived from one of the curves of Figure 4 with the 15% scatter spreading the points, and Figure 2 from one of the curves in Figure 3. The cryptic parentheses in Figures 1 and 2 can now be deciphered.

Recall now that the point of all these observations is to find a robust way of assigning the parameters of equation 8. Recall also that we have assumed β to be a known parameter, historically continuous, so that if our data deviate from expected ranges of β (particularly, on the low side in absolute value) we can suspect an approach to a minimum. Finally, recall that at any given time A(t) is an arbitrary normalizer. In other words, all these observations are aimed at deriving P_{Min} and δ.

Now note that if δ is small, P_{Min} is rather sharp, and should be quite easily observable. Unfortunately, this does not often actually happen. For example, referring to Figure 3, if the scatter of points is ±15%, we might have difficulty locating P_{Min} within a factor of at least 2, even for δ as small as 0.1. A fairly robust rule for locating P_{Min} <u>when the existence of a minimum is clearly visible</u> is to estimate it as

$$P_{Min} \doteq P_1^{1/3} \; P_2^{1/3} \quad , \tag{14}$$

where P_1 and P_2 are, respectively, the lower and upper end of the range over which P_{Min} appears to lie. Given P_{Min}, δ can usually be estimated fairly precisely, and if older estimates of δ exist it can be compared to these estimates.

A more troublesome case is to try to locate P_{Min} when one can not see it--or at least not see it clearly. There are two sets of circumstances under which this problem might arise:

1. When δ is large, and one sees an essentially flat range of P over which variations of C are lost in the data noise.

2. When the data do not extend up to P_{Min}.

The second of these cases is fairly rare, for reasons we shall discuss later. The first case does permit fairly robust data estimation, however.

When the curve shows a flat bottom, we have a good estimation of

$$C_{Min} = AP_{Min}^{-\beta} (1+\delta) \quad . \tag{15}$$

For example, if we take data within a scatter range (which should be visible) we can isolate those data within 2 of these sigmas of some minimum point. C_{Min} can be roughly estimated as the value below which 1/3 of these data points lie, and its value is unlikely to be as much as one sigma in error. This gives us one estimate of a function of P_{Min}, δ (assuming that A, β are known from extrinsic data). We then need another functional relationship to be able to estimate P_{Min}, δ.

Of course, the ability to estimate P_{Min}, δ is of somewhat academic interest in this case. What we know is that C is close to C_{Min} for a considerable range of P. Under these circumstances, very temporary or local considerations--the size of an available operating labor force, the compatibility of the new plant with the existing industry structure, or very poorly defined evolutionary potentials of plants of the size chosen--will dominate design specifications. Nevertheless, one is interested in extrapolating, and for this, knowledge of the parameters is needed.

Engineering--Psychology and Practice

Under two circumstances, just listed, it becomes very difficult to determine the scale parameters of engineering systems such as industrial plants. To these, we may add the third circumstance, that occurs when the industry is new and there are simply not enough data.

Twenty years ago, we would have had to admit defeat. However, we have gotten used to Bayesian estimation in this generation. This essentially amounts to utilizing the data processing ability of the human brain to arrive at reasonable answers to questions, even when complete input data are lacking and the reasoning program can not be specified. In one way or another, the method reduces to consulting the experts; in this sense, experts are people with good knowledge of the system under consideration and similar or analogous systems. (Preferably, also, experts are people who have records of good estimation in other, unresolvable problems; but this is a very hard qualification to check.) For questions of industrial scaling these experts come from the engineering community.

It is therefore proposed here that one look at engineering practice for clues to the problem of "unknown scaling". But one has to understand this practice to interpret it. I claim the right of a physical scientist, more or less drafted into the ranks of engineers, to perform such an interpretation.

Among other things, engineers, and particularly those engaged in project design, are goal-oriented. The basic ethic of engineering is to use the forces of nature to serve human goals. What the goals are is usually considered extrinsic though; they are set by the client. (Engineers are in consequence always mystified when these goals are not merely challenged, but blamed on them. And there's truth in this bafflement. Why doesn't someone blame the business and political leaders? And don't they get their goals from the humanists and social scientists?)

An important goal is economics in the engineering sense: the product is to be provided at minimum cost to the producer. (The engineer does not question if a social cost is externalized. Somebody made the rule and it is accepted. Engieneering is a law-abiding profession, and also accepts rule changes which internalize social cost. It makes for a new design job. Within the new rules, one also looks for minimum production cost. What frustrate engineers are rules that provide process, rather than performance specifications. Sometimes these actually frustrate achievement of the desired performance goals, be they pollution abatement, "buy domestic", "create employment" or whatever. Sometimes they merely require expense for little result. But these are only surface manifestations of a greater discontent. The engineer's job is to find and design the process that achieves the performance goals, and when the process is mandated it means that some "amateur" has struck at the core of his professional pride.)

Engineers are also conservative. There is a strong tendency for this to be interpreted in the political sense, where in fact it also applies; but this is actually a coincidence. The basic conservatism, though is the tendency to be pessimistic about change. It interferes with the ability to expect results with confidence. Murphy's Law was invented by engineers, from bitter experience. Thus, engineers are accustomed to taking small steps. They prefer new designs to evolve incrementally. The "back to the old drawing-board" joke (describing the engineer's comment as the airplane crashes during the test flight) is approximately 100% wrong in describing engineering attitudes. (In the early 1950's, Enrico Fermi was appalled by the slow progress of nuclear energy. He attributed this to the engineering conservatism which had already taken over. His aphorism was to the effect that "What we need are some reactors that don't work".)

Since one of the goals of project design is economics, what we are dealing with is more than a tendency. The engineering team will simply not perform a job that doesn't fill the bill; or, if ordered to do so, they will make sure that the customer knows that they think it's stupid. If they are convinced that the job is uneconomic, there are many ways of making the cost look a little worse; and believe me, the tricks are constantly used.

With this background, it is clear that, on the one side, engineers prefer the conservative and the incremental, whenever possible, and do not like "great leaps forward". This argues for a tendency to always keep plant sizes below the point of minimum cost.

But there is another side...

Engineers are quite naive in trusting rules of thumb, and the scaling law is one such rule that is generally credited. The law that is believed has the simple form of equation (1). Indeed, this is a major source of the belief that the technologist always is looking for "bigger and better" with the assumption that bigger _is_ better. So, countering the generally conservative outlook is the notion that economy comes in the large, economy size. The design group will always look longingly at the largest plant that can be built; they will insist the only obstacle to continued economies of scale is the inability to go further in size, in case existing manufacturing or construction methods have limits.

It is my contention that conservatism and naivité lead to a balancing of mystique, and that, given this balancing, _the actual rating of units delivered at a given time is a good estimate of P_{Min}_. In a sense, it is suggested that, when faced with producing equipment, the engineering community performs a valid Delphi analysis, and balances its prejudices for the big against its prejudices in favor of existing practices to achieve a result that is relatively free of bias. Indeed, my discussion of engineering psychology had the purpose (besides the obvious one of blowing off some steam) of indicating why bias ought not to be pronounced in these scaling decisions.

Scaling and Learning Parameters

Up to now, I have asked you to absorb a great deal on pure faith. It may be summarized as hypotheses to be tested. This testing can not lead to validation of the hypotheses in the sense of proving their correctness; but to another type of validation--that of utility. In the following section I will be studying the parameters of nuclear systems under two hypotheses:

-- First, that the mean size of new systems, as measured by electrical capacity, is a good approximation to the optimal size, P_{Min}.

-- Second, that the optimal size, P_{Min}, is subject to a continued learning process, such that P_{Min} increases in a power law as the experience of an industry grows.

From these data, one can infer some conclusions, that may or not be correct, about the future technical evolution of the nuclear industry.

NUCLEAR SCALING

The Approach

We are now ready to apply the hypotheses of the previous sections to data. The basic data that we use are those that describe the electrical capacities of nuclear power plants as a function of their sequence of commissioning. In other words, we are using the "hardest" possible data--an unmistakable and unfakable statement of electrical power capacity--to derive the parameters of the scaling-learning law, equation (10). The data are those collected by NUCLEAR NEWS in their most recent World List of Nuclear Power Plants [4].

Light Water Reactors

Figure 5 exhibits the data for P vs. n, plotted in a way that requires a bit of explanation. The scales are P, electrical capacity, and n, sequence of commissioning. However, individual reactors are not plotted. Instead, the P number (ordinate) represents average power of reactors commissioned in a given year and the n number (abscissa) is the geometric average of the sequence number of those reactors. The logarithmic plot is used because a power law is being investigated. Finally, the data are for all reactors, not just LWR's. However, so dominant are LWR's that we can take this as an LWR curve without significant error. The points marked by circles are reactors already commissioned and those marked by squares are those that are planned.

The straight line is a regression of these individual points. The regression analysis is not too sound theoretically, as the different points represent different numbers of reactors commissioned in different years. However, the line looks good enough so that it is unlikely that weighting would change the results.

The equation of the line is

$$\log P \doteq 2.01 + 0.385 \log n \quad , \qquad (16a)$$

or

$$P \doteq 100\, n^{0.385} \quad . \qquad (16b)$$

The result of the fitting suggests the following:

- -- Reactors commissioned in 1966, 1967, 1968 and 1969 were probably below the size of minimum unit cost in that era. These were reactors ordered in the early 1960's. By 1970, the optimal installed size of reactors was around 530 MWe, and roughly corresponded to the "Connecticut-Yankee, Oyster Creek" models ordered in the mid 1960's.
- -- The first reactor considered "commercial" should have had a capacity of about 100 MWe.
- -- As of now (1979), the optimal reactor size is about 830 MWe.
- -- By 1990, the optimal size will be around 1100 MWe.

General knowledge of the nuclear industry indicates that the gradual increase of commissioned size stems from variations in the mixture of 600, 900 and 1200 MWe (approximately) designs--the "standard" sizes--orded in the 1970's. The largest ones were those most commonly delayed, and therefore dominate the later commissioning dates. There is a suggestion (although it must be conceded that the reasoning is tautological) that the largest-size LWR's were ordered prematurely; that is, when these largest sizes were greater than P_{Min}.

Using a β of 0.5, equation (13b) suggests that b is of the order of 0.2. Learning is not rapid in the LWR business. Many in the industry blame this fact on the impact of ever-stricter regulations: that a lack of resilience to regulatory change is a principle reason for cost minima developing in the nuclear business.

The data available to me do not permit an estimate of δ, but John Fischer's curves [3] indicate that the minima are rather flat. This observation corresponds to large δ, and large δ means that

- -- It doesn't make too much difference what size is ordered, the unit costs are about the same.
- -- The plants are costlier than disciples of the simple scaling law expect.
- -- There is a tendency to go past the minimum for planners who have a "bigger is better" philosophy; they are not severly punished after all.

These qualitative observations seem to me to fit the LWR industry quite well.

Heavy Water Reactors

Figure 6 presents all the heavy water reactors in the world, plotted one-by-one. The regression line is drawn in, but it is probably meaningless; for the points seem to fall into two groups. The higher group is essentially Canada and the lower group is most of the rest of the world. For the record, the regression line is

$$P \doteq 65\ n^{0.628} \quad . \tag{17}$$

Because the data from world HWR's did not give good grounds even for trying to fit a power law, the data were plotted for Canadian pressurized-heavy-water-cooled HWR's only. These are plotted in Figure 7. As with Figure 5, crosses are history and circles are expectations. Regression of these data gives

$$P = 312\ n^{0.318} \quad . \tag{18}$$

The rms scatter of P about the formula is not bad - it is very close to ±20%.

As with the LWR data, the scaling-learning law for Canadian heavy-water reactors likewise appears reasonable. Economies of scale are initially large for such reactors, since at small sizes the core inventory of expensive heavy water is a forbidding item of cost; as power increases, the inventory of heavy water per unit power drops toward an asymptote. It is not too surprising, therefore, that the initial size of commercial units should be greater for HWR than for LWR. However, the actual power of the "demonstration" unit, the first commercial reactor to be built, was approximately 200 MWe rather than 300 MWe. Again, as with early LWR's, there are indications that during early commercialization undersized systems were used. The philosophy of "cut your losses while you are learning" appears to be evident, and it is logical.

We have argued that HWR's should have larger β's than LWR's; on the same basis as we postulated a b of about 0.2 for LWR's we would conclude that b is about 0.15 for HWR's. The rate of "learning" seems to be quite small (slow). Canadian regulatory practice has been far more consistent than U.S. practice, and this argues against regulatory change as a prime reason for slow learning. It also casts doubt as to whether this is the reason for slow learning in LWR's. It may be that for all nuclear systems, the requirements for engineered safety and emergency subsystems become more demanding as capacity increases, and that the complexity of these subsystems becomes even greater as a result.

The Canadian case also illustrates a balancing between economies of duplication--pure learning--and economies of scale. Stations come in sets of four units, which are then often augmented by a second set of four units that are only slightly different in design and rated at essentially the same power. The middle units of these groups of eight come closest to the regression line. It is obvious that a charge in unit capacity represents a one-time cost for redesign, and that it becomes increasingly more economic to defer this cost. Only when the new-economy of scale, from scale-learning, can pay for redesign, does it make sense to do that job. The prediction on that basis is that, in Canada the 880 MWe units of the Darlington station, which are the last four circles in Figure 7, will be standard design for quite a long time.

Remarks on Fast Breeder Reactors

There have not been enough Fast Breeder Reactors (FBR) built or contracted in the world to make a judgment on their scaling laws, but the following points are pertinent to what would be a logical course for their development:

- -- They should have rather large β values: their design costs are high and design practices are evolving, and this always moves β towards a value of 1.0.
- -- They are not now economic, and will not be until two things have occurred: achievement of standard designs; and development and operation of full-sized fuel cycle facilities to service them.
- -- Their complexity seems to increase fairly rapidly with unit size, a point which argues in favor of a fairly sharp minimum (i.e., a fairly low value of δ).

Fast breeders of the world are listed in sequence in Table 1.

It can be argued that the "standardization of unit sizes around 300 MWe represents the effect of many independent developments going on during a "cut the losses" period. It certainly appears that BN-600, which has suffered many delays, is at the upper limit of what might be expected of a second national reactor. Super Phenix is an early prototype of what is hoped to be a standard commercial size later, but will almost certainly be a money loser compared to smaller units now.

By analogy with thermal reactors, I would expect that the maximum learning and best economy of the next generation of FBR's will occur for designs in the 500-600 MWe range, and these should be standard for some time to come. The mystique of size is probably premature, even for reactors designed now. More learning (and more investing in learning) still is needed.

Summary of Observations

It now appears that the quite large unit sizes adopted for LWR's in the early 1970's were premature. However, the penalties were not great, because it is almost certain that the cost-minimum power range is quite broad. In brief, one might expect that for any individual station, the local and temporal factors of convenience in financing, procuring and operating LWR's should govern the choice of plant ratings far more than the actual capital cost economies. These remarks hold for units (probably) between 300 and 1500 MWe in rating.

The development of Canadian HWR's is more orderly and best illustrates the gradual evolution of standardized designs as learning proceeds. The Canadian experience also might be used to estimate the one-time costs of redesign for adoption of larger ratings as standard. HWR's seem to feature even slower "learning" than LWR's, whose learning is already rather slow. There are hints that the region of minimum unit cost is less broad--i.e., the parameter δ is smaller--for HWR's than for LWR's, so that it is more important for HWR's to be scaled properly.

By analogy, FBR's would be expected to exhibit still sharper cost minima than HWR's, and there will then be a large penalty associated with going overboard on "bigger is better" philosophies. FBR's, with one--perhaps two--exceptions, are still being built in "demonstration sizes", however. These are units that are deliberately smaller than the most economical rating, chosen because even the most economic size is not competitive during the period of market entry. Their ratings are around 300 TWe. Thus, we can probably guess that units around 600 MWe will be the first "economic" size in the sense of being the first to provide electricity at costs competitive with other power generating systems. Again by analogy with LWR's and Canadian HWR's, the phenomenon of scale-learning is likely to be fairly slow, and the medium-sized breeder will probably be with us for a long time.

A POSTSCRIPT ON PLANT SIZE AND INDUSTRY SIZE

I have argued that the phenomenon of scale-learning, the increase in plant rating that corresponds to minimum unit cost, as experience is gained, is a major factor in unit size growth. I have also argued that this factor is essentially autonomous; that it depends on plant characteristics and characteristics of the plant construction industry.

It has, however, been often noted that plant size tends to increase roughly at the same rate as the industry with which the plant product is associated; i.e., that simple volume of business is an important factor on plant capacity.

I find this to be intellectually disturbing. After all, if an industry doubles its volume of business, it is still possible that two plants on one site will be a better manufacturer's response than making a single larger plant. Obviously, distribution limits on the product are a factor--a strong one in the electrical supply industry. However, at least as regards nuclear energy, scale-learning seems to be slower than industrial growth. This also seems to be the case with coal-fired power plants [3].

I would therefore like to offer, as a devil's hypothesis, that the ratio of plant to industry size is just one more factor in scale-learning, and only rarely a dominant factor. One then has to attribute to coincidence the many correlations that have been documented. I would suggest as an object for future study that this coincidence has been reinforced by prejudicial definition: when are 2,3,4...n units on a single site a single plant or n plants?

Table 1. Fast Breeder Reactors

Year	Country	Name	Rating (MWe)
1963	U.S.A.	Enrico Fermi-1	57
1973	U.S.S.R.	BN-350	350
1973	France	Phenix	233
1976	U.K.	Dounreay PFR	250
1980	U.S.S.R.	BN-600	600
1983	France	Super Phenix	1200
1983	F.R.G.	SNR Kalkar	300
1985	Japan	Monju	300
Under debate-deferred	U.S.A.	CRBR	350

Figure 1. A Posteriori Estimation of Scaling Law

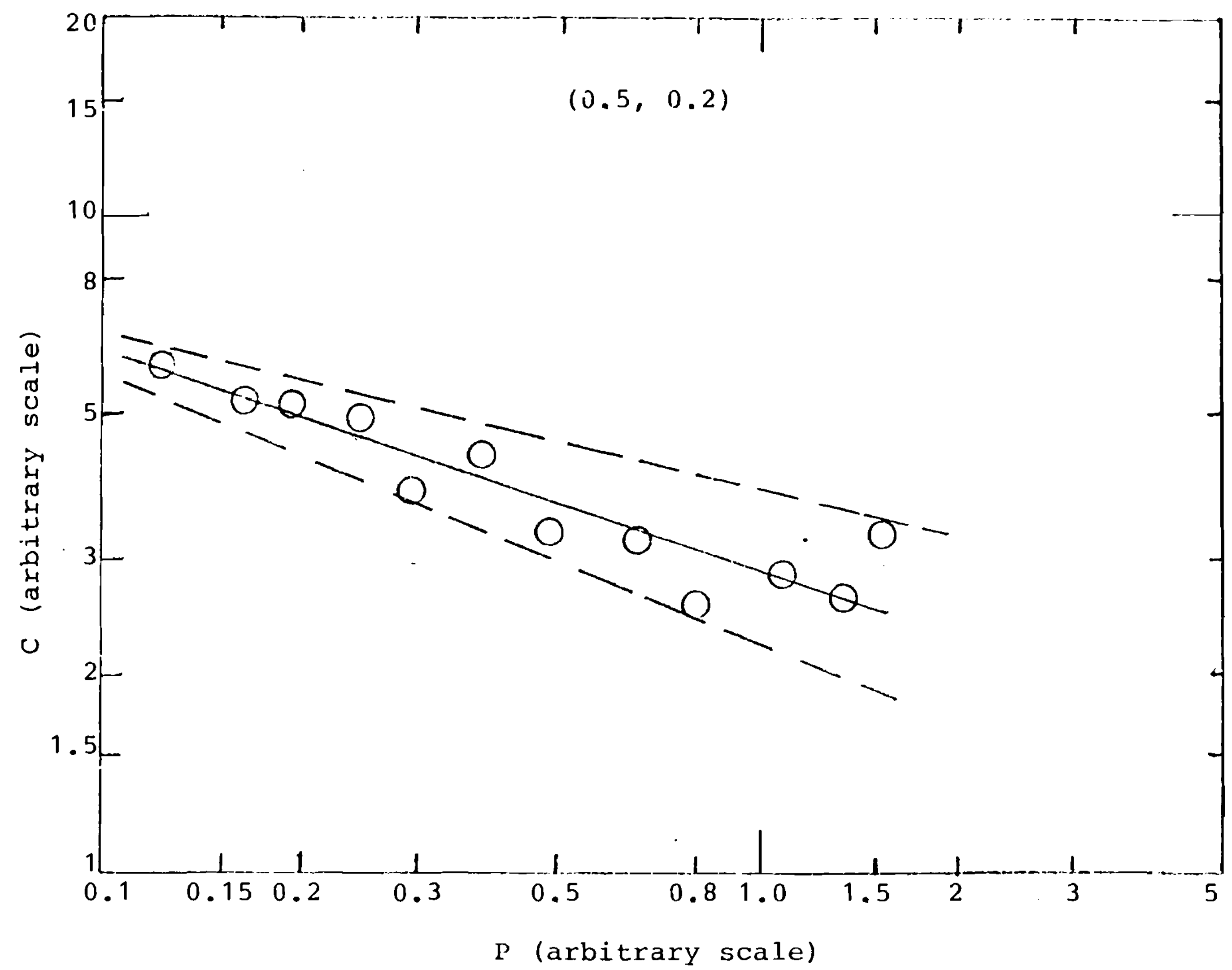


Figure 2. A Priori Estimation of Scaling Law

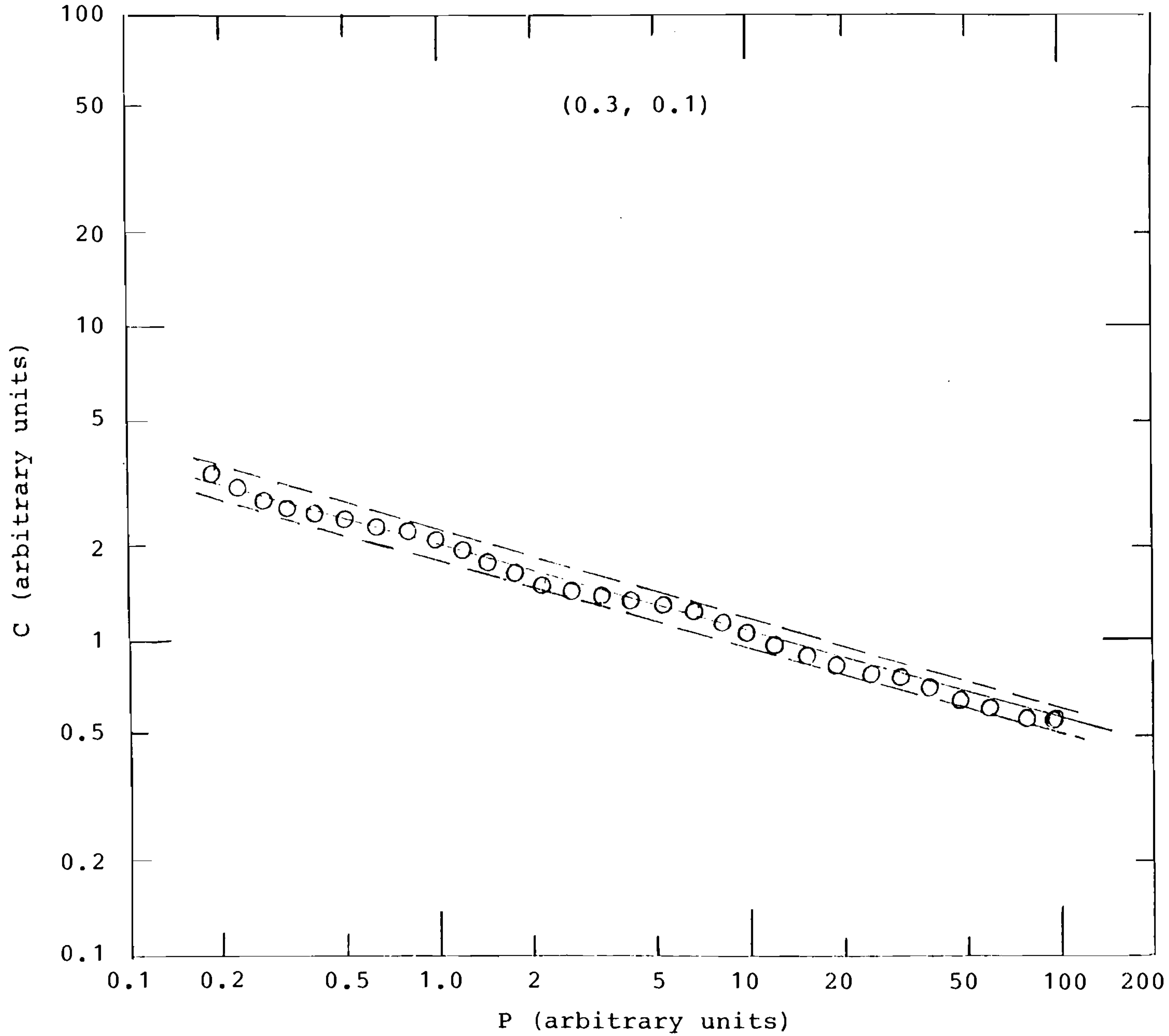


Figure 3. Complex Scaling Law for $\beta = 0.3$

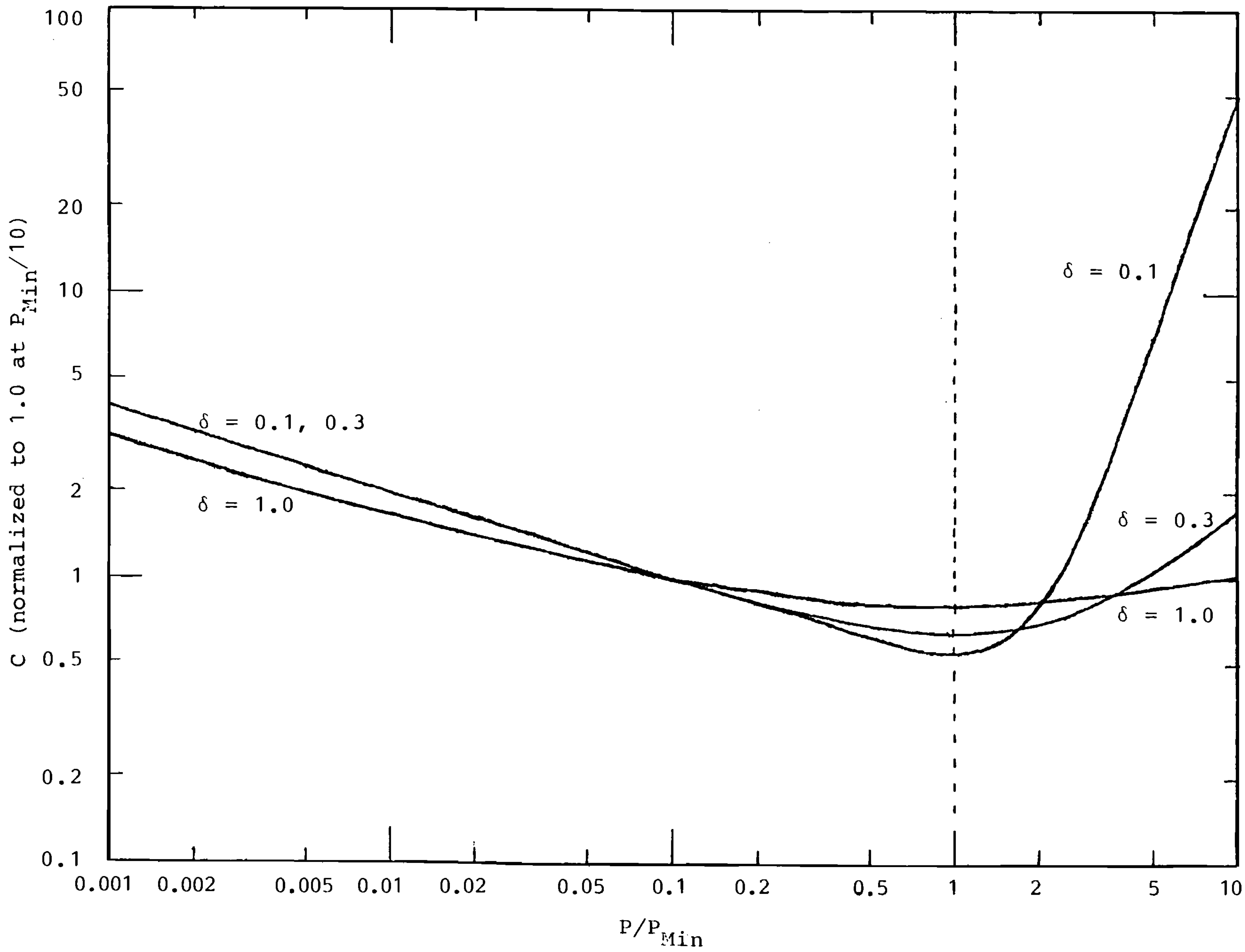


Figure 4. Complex Scaling Law over a Limited Range of P

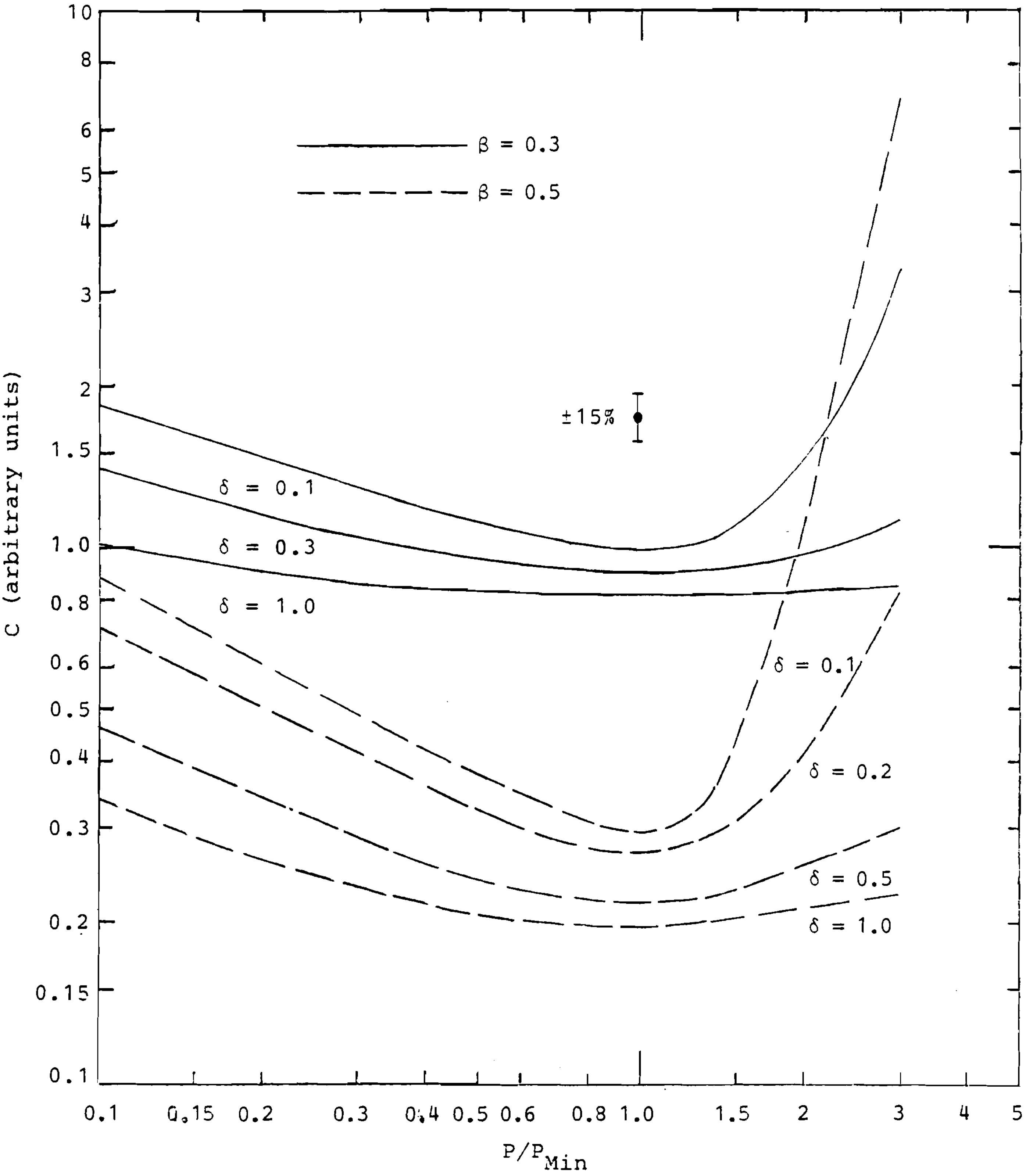


Figure 5. Annual Average P and n for Power Reactors (Mainly LWR's)

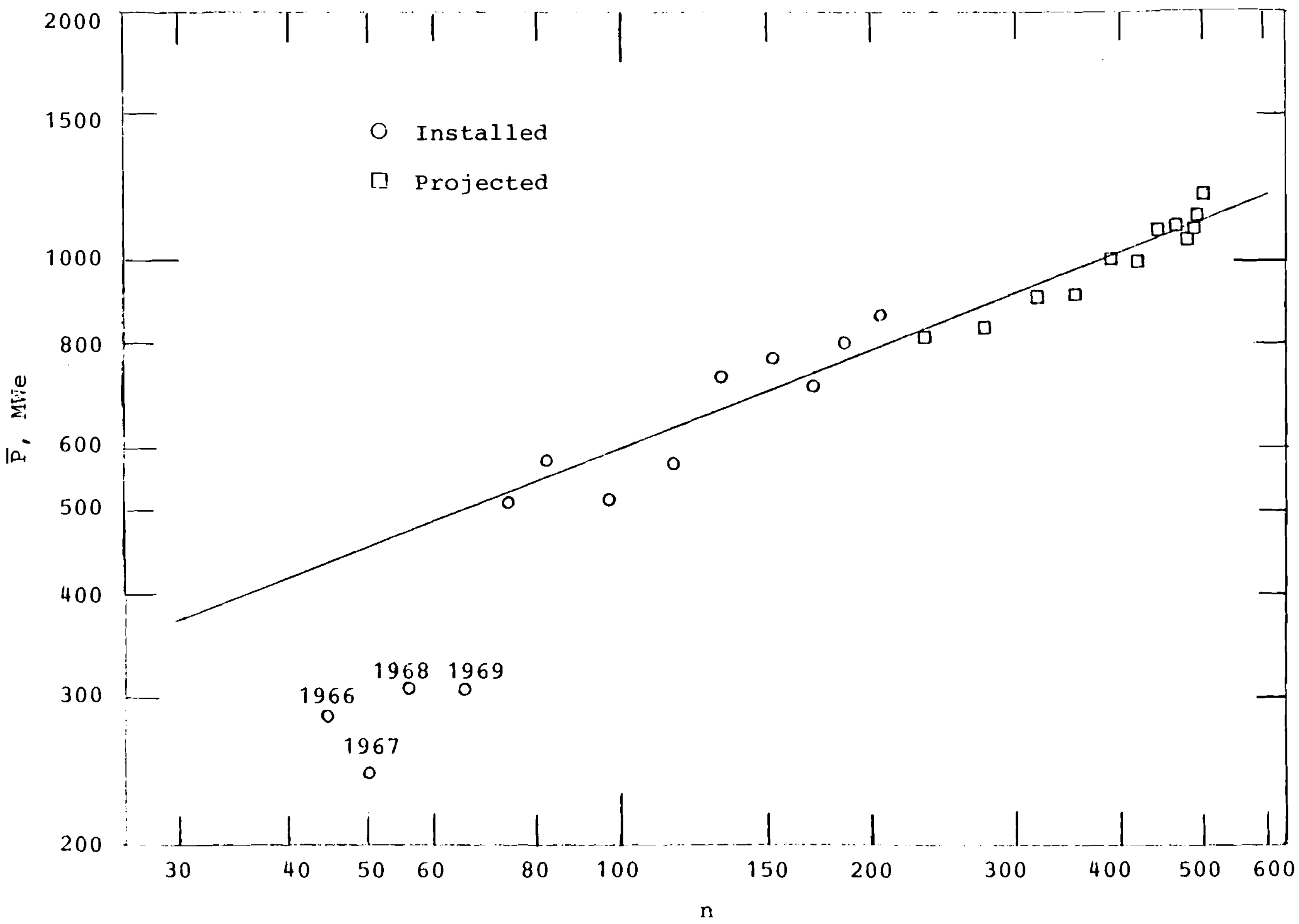


Figure 6. P vs. n for HWR (World)

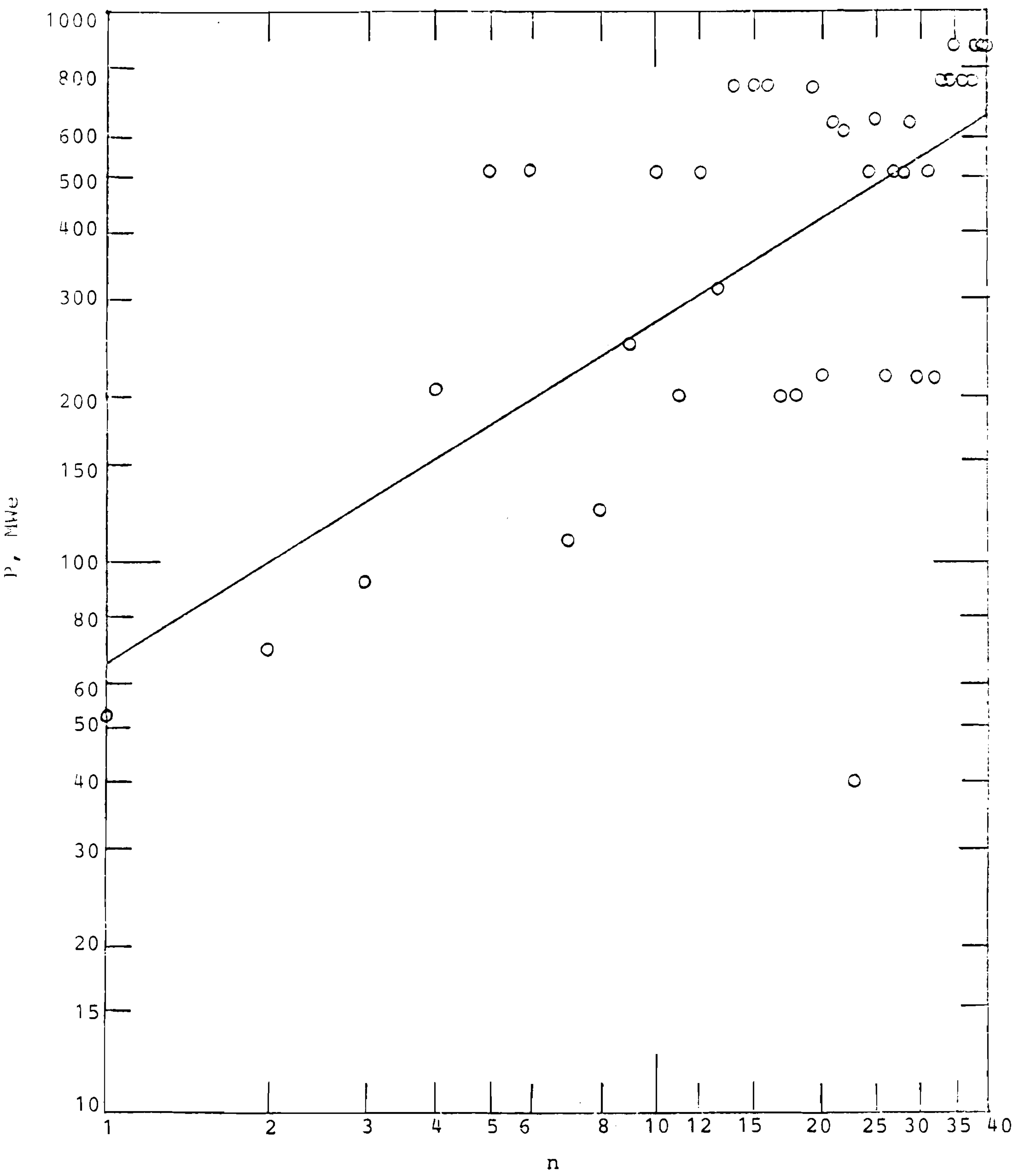
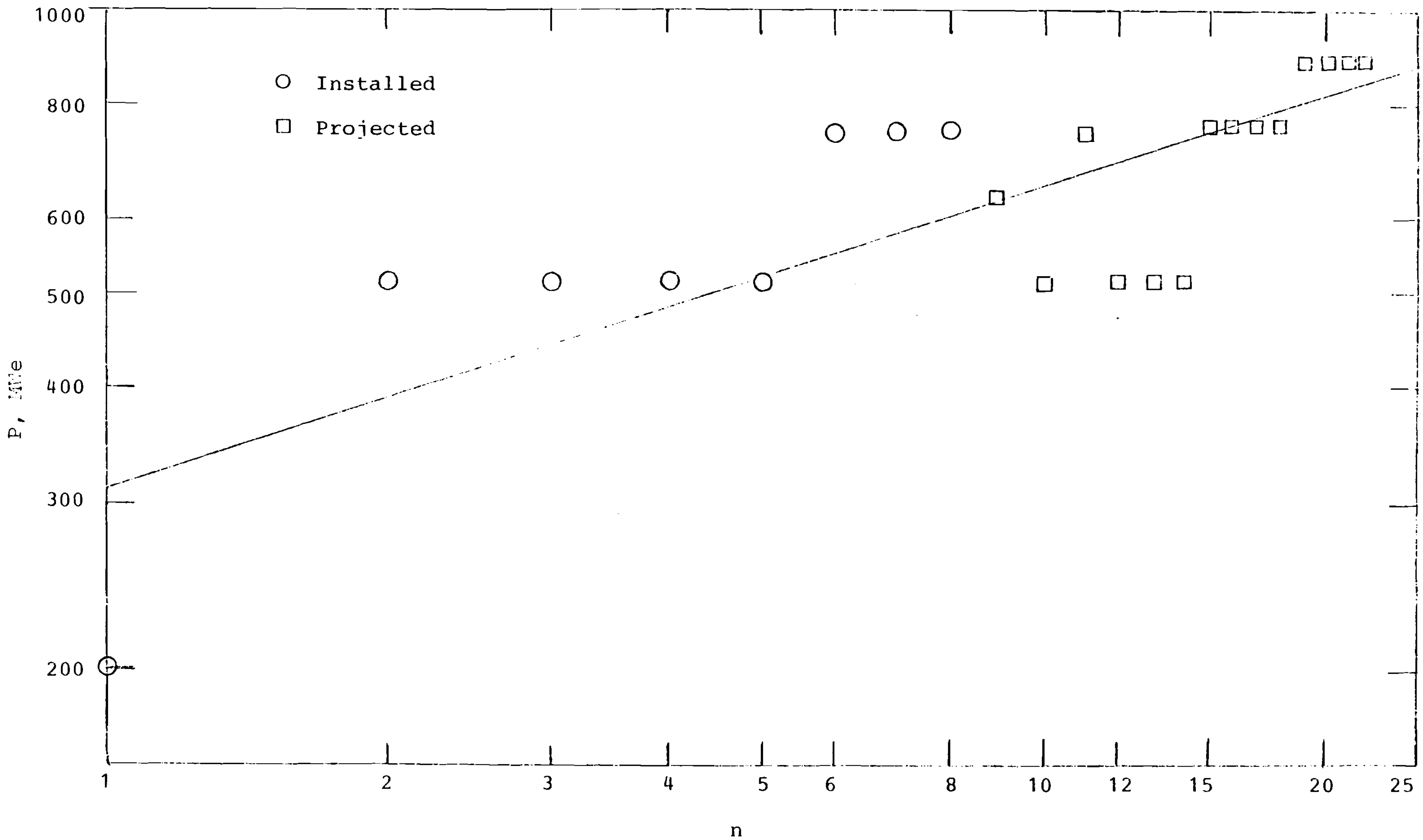


Figure 7. P vs. n for HWR (Canada)



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